Concept Generation and Computational Techniques Applied to Design for Transformation

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Transformers are a class of products that exhibit a change in state to facilitate new or enhanced product functionality. The historical children's toys known as "transformers" provide a mental picture of this definition. Working examples are vertical lift aircraft that function as helicopters for take-off but transform to propeller-driven airplanes for point-to-point travel. In this paper, principles and design methodologies are introduced for the creation of transforming products. Also, an approach is demonstrated for implementing Transformer Design Principles as part of an ideation and computational design process. An application to an Unmanned Aerial Vehicle (UAV-TACMAV) illustrates the utility of the approach.

Nomenclature

L = lift

 C_L = lift coefficient ρ = density of air

V = velocity of wing relative to free stream velocity

A =area of wing h =wing camber c =wing chord

 α = angle of attack of wing relative to free stream

D = total drag on wing $C_D = \text{drag coefficient}$

 $C_{D,min} =$ minimum drag coefficient AR = aspect ratio of wing e = Oswald's efficiency factor $L_{min} =$ minimum acceptable lift $D_{max} =$ maximum acceptable drag

I. Introduction

transformer is a system that exhibits a state change in order to facilitate new or enhance existing functionality.

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Transformational design facilitates the development of products with a much broader functional repertoire than traditional single-state designs. Current design theory lacks a systematic methodology for the creation of products that have the ability to transform [1, 2]. A methodology is currently in development that enables organized and reliable creation of transformers. The methodology involves a set of principles and facilitators governing transformation and an associated process for applying them to design problems. Towards this end, an ideational and computational approach to implementing transformer theory is outlined in this paper. To illustrate the approach and demonstrate its utility, the transformation theory is applied to a specific design problem: the design of a lightweight, quickly deployable, easily operable, compactly storable wing for a micro aerial vehicle.

Following a general description of transformational design methodology, several transformational design principles and facilitators are listed and discussed in general terms. Then, they are applied to a design problem and used to create several concepts. The preliminary concepts are evaluated and one concept is selected for development. This idea is embodied as a CAD model which is then computationally optimized for performance.

A. Motivation: Transformation

The state changes that occur in transformers may occur in a variety of domains (chemical, electrical, etc.), but for the purposes of this research, only mechanical transformation is considered. Mechanical transformers exhibit a change in physical dimension and/or form to perform separate functions or enhance an existing function.

Transformational design greatly increases the ability of the designer to create products that satisfy a broad range of customer needs. Multifunctional devices can be designed that perform their tasks with greater efficiency than traditional devices, and functionality can be obtained that was previously unavailable.

Devices exhibiting transformational qualities currently exist, but a systematic methodology is needed to facilitate the development of such devices. Such a methodology provides designers with a set of comprehensive design principles and facilitators that can be used to produce transforming devices, and a process by which this transformation theory can be applied. Product platforms and deployable structures are described for the purpose of stating distinctions and similarities against products that transform

B. Product Platform Architecture

A platform can be defined as a common set of physical or non-physical modules from which multiple products can be derived [3]. Changes in functionality among derivative products can be achieved, for example, by changes in the modules that are attached to a fixed platform. Previous work defines and describes product platforms [1, 4, 5, 6,

7, 8], and tools exist that help identify modules and platforms when designing such systems. Product platforms are architected during the embodiment phase of a product and help create product variants during its manufacture. For example in spacecraft development there are potential advantages in reusing common subsystems and components and developing multiple missions based on a common set of sub systems and components [8]. A platform is used to help design variants of a satellite or a spacecraft, when the cost of making totally different designs and models would be prohibitively expensive (Fig. 1). Whereas product platforms are adjusted for customized functionality during the product development and manufacturing phases, transformation refers to a change in product functionality during the *use* of the product. Transformational design would involve, for example, the design of a spacecraft or a system of the spacecraft to transform itself, adjust its functionality, and deploy as a satellite.

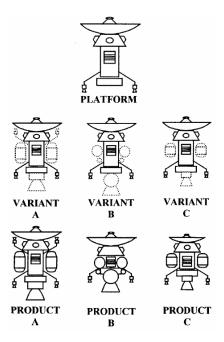


Figure 1. Platform base product family [8]

C. Deployable Structures

Deployable Structures are structures that are primarily used in their deployed or expanded configuration and don't serve a function in their collapsed configuration except for saving space for storage. Deployable structures have existed for a very long time and have brought about innovation in the design of products and structures alike. These deployable structures ingeniously expand and collapse and have solved a number of challenges that design engineers have faced. Deployable structures are being used everywhere from roof structures (Fig. 2) to aerospace applications (Fig. 3 and 4). Collapsible products like the umbrella are used everyday by many people. Collapsible

systems are captured in our definition of transformation, as they exhibit a state change to facilitate a new functionality of stowage or deployment. Deployable structures or collapsibles are used primarily in one configuration and designed to collapse for storage or concealment as illustrated by the examples in Figures 2, 3, and 4. The concept of deployment may be an integral part of transformers, but to qualify as a transformer, the product or system must exhibit different functionality between expanded and collapsed states or during the transition between the states.

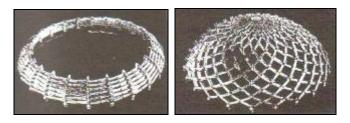


Figure 2. The Iris Dome (Hoberman 1991) [9]

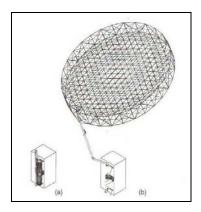


Figure 3. Astromesh reflector; (a) stowed; (b) deployed [9]

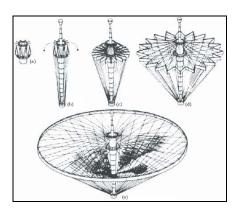


Figure 4. Deployment sequence of the Hoop / Column Antenna [10]

D. Advantages and disadvantages of transformation

Transformers do not need to be complex to achieve a change in functionality. For example the ladder-chair shown in Figure 5, quite easily changes between a chair configuration and a step ladder configuration.

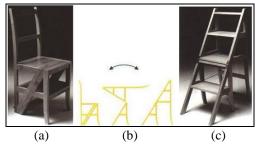


Figure 5. (a) Chair configuration; (b) transformation; (c) ladder configuration [11]

Transforming devices hold many advantages over single-state designs. The primary advantage is that the same device is able to perform multiple functions. By creating a transforming product that performs the tasks of multiple conventional devices, efficiency can be increased across a wide range of customer needs. For example:

- 1. Multifunctional benefits may be achieved by designing one system to accomplish the functions previously performed by separate products. For example the Bell Eagle Eye (Fig. 6) is a Vertical Short Takeoff and Landing (VSTOL) Unmanned Aerial Vehicle (UAV) which has a tilt-rotor system that allows it to vertically takeoff and land, while also cruising at fixed-wing aircraft speeds [12]. These types of aerial vehicles essentially incorporate the benefits of vertical takeoff and fixed wing flight.
- 2. Manufacturing costs may be reduced by utilizing less labor and material for a single device.



Figure 6. The Bell Eagle Eye UAV [12]

3. System mass may be lowered for weight-sensitive applications. For example having a 6 in 1 screwdriver tool (Fig. 7) that replaces 6 single screwdrivers decreases the burden of carrying six separate screwdrivers. The Lock n' Load screw driver contains 6 screw bits that can be changed by pulling the handle backward, twisting it and pushing it back in again.



Figure 7. Lock n Load 6-1 screw driver.

4. Certain transforming devices may perform functions between states that are not possible in single-state products. For example SHIFT (Fig. 8), an innovative bicycle created by Purdue University [13] is a new trike-

bike design that lets kids learn to ride on their own by giving them three-wheel stability at low speeds with the balanced freedom of a two-wheeler at high speeds.

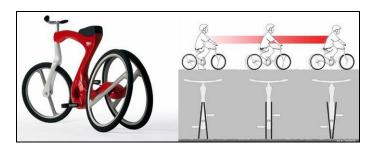


Figure 8. SHIFT's wheels shift as the child gains momentum and learns to balance [13]

- 5. The whole is greater than the sum of its individual parts. A single design that can transform to perform multiple functions may have an increased functional repertoire compared to several single-state devices.
- 6. The functions that a single product can perform need not be related or require the same structural layout, as with other single-state multifunctional devices. For example, the Unmanned Switchblade due to be completed by 2020 is a shape changing concept of an unmanned aerial vehicle under development by Northrop Grumman [14]. Figure 9 shows the Switchblade swiveling its wings by approximately sixty degrees before breaking the sound barrier. The claim is that this reconfiguration redistributes shock waves that accumulate in front of a plane at mach speeds and induce drag. At sub sonic speeds the Switchblade's wings swivel back so that they are perpendicular to the fuselage like a normal plane. The Scorpion / Manta UAV (Fig. 10) utilizes a free-wing concept, which allows the outer wing panels of the aircraft to operate at a constant angle of attack with variable incidence, exactly opposite that of a normal aircraft [12]. The tilt-body creates vectored thrust to obtain very short takeoff and landing, and the free-wing allows operability in gusty conditions, less need for sensor stabilization, and increased airframe life and decreased airframe weight due to reduced gust loading. In essence the free-wing brings about a function change to the whole UAV by making it a VSTOL UAV and giving the UAV a constant angle of attack for varying angle of incidence, which helps in gust mitigation.

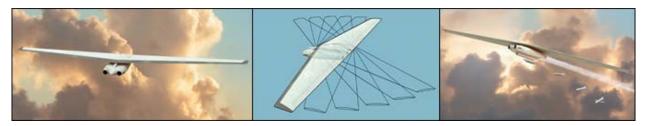


Figure 9. The Unmanned Switchblade Concept under development by Northrop Grumman [14]



Figure 10. The Free-Wing Scorpion / Manta UAV [12]

Transformation does, however, carry potential detriments to the design process. One of the goals of a design methodology is to reduce or eliminate these disadvantages. The most prominent issues are:

- Transformational design requires significantly more initial time to develop successful products. Due to the
 complexity of transformers in general, their design produces unique technical issues that must be overcome.
 The additional cost associated with their design must be weighed against their functional benefits.
- 2. The inclusion of transforming elements can increase the redundancy of a device when the device is used ofr only one of its functions. Again, these must be considered in light of the added functions provided by reconfigurable devices. For example take the 6 in 1 screwdriver (Figure 7) versus 6 individual screwdrivers; a person may only want to use one particular screwdriver but in the former case he/she is carrying a heavier and more voluminous tool compared to a single screwdriver.
- 3. With any multi-functional device, certain functions may be impacted negatively by the inclusion of others. As components in a multi-state multi-functional Products (transforming products) are frequently shared, their design is governed by two or more separate sets of design parameters, which may or may not be aligned. In order to accommodate the elements necessary for transformation, tradeoffs are inevitable where the design requirements of one state constrain the design requirements of another state in the transforming product.

These potential problems posed by the design of transformers must be addressed in the creation of a formal design methodology. Although they pose obstacles in the path of a successful design, they are not insurmountable, and tools can be created to help overcome them.

II. Transformation Design Theory

A. Transformation Principles

A methodology for the design of transforming devices includes a set of principles that define transforming devices, and a collection of facilitators that can be used in their design. Collectively these principles and facilitators constitute a developing theory of design for transformation. Accompanying tools will be used to apply these to the design of transformers. Research has produced a number of principles and facilitators that can be employed to aid in the design of transforming devices. They were derived primarily through an inductive process that includes studying patents, products and natural analogies. This principle development process as well as a comprehensive description of the theory is described in a separate work [15]. A subset of these principles and facilitators follows.

Transformation principles are generalized directives to bring about a certain type of mechanical transformation.

They are guidelines that, when embodied singly, create a transformation. Some examples of transformation principles include:

• Fuse/Divide – Make a single functional device become two or more devices, at least one of which has its own distinct functionality defined by the state of the transformer or vice versa. Several different individual parts or systems (which may or may not have distinct functionality) can combine to create a single separately functioning device. Similarly, a single functioning device or part may also decompose into two or more components. The memory module from the mp3 player in Figure 11 detaches and becomes a USB drive.



Figure 11: Creative MuVo USB 2.0 [16]

• Expand/Collapse – Change physical dimensions of an object to bring about an increase or decrease in occupied volume along an axis, in a plane, or in 3-dimensions. A device is designed to expand and/or collapse, allowing for storage or altered functionality depending on the state of the device. Expansion/collapse can embody

combinations of different mechanisms and/or flexible/elastic materials. Figure 12 shows a firefighter's ladder, manufactured by Gunzberger Steigtechnik, which also functions as a battering ram in its folded state [11]. It is a transforming device that uses the expansion/collapse principle.



Figure 12. Firefighter's ladder transforms into a battering ram [37]



Figure 13. Puffer Fish [17]

Nature has its own share of transformers that use expansion/collapse to alter functionality by altering their configurations. Figure 13 shows a puffer fish which uses the flexibility of its skin as a membrane, which expands when it fills its body with water, to intimidate enemies. The wheel spider shown in Figure 14 collapses its legs towards its body to transform into a wheel structure to roll down sand dunes rather than crawl down.

The bag in Figure 15 expands from a towel to a tote bag when the strings on periphery of the towel are pulled outward. In the tote configuration, the strings function as the shoulder straps.

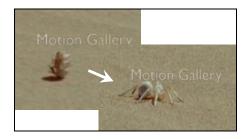


Figure 14. Wheel Spider [18]

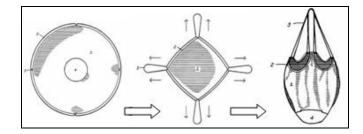


Figure 15. Collapsible Hand Bag / Towel [19]

B. Transformation Facilitators

A facilitator is a design architectural feature that helps or aids in creating mechanical transformation. They aid in the design for transformation, but their implementation does not create transformation singly.

• Shared power transmission – Transmit power from a common source to perform different functions in different configurations. Power is transmitted from a single power source (i.e. an engine, motor, etc.) to perform separate

tasks in different states. The OspreyTM shown in Figure 16 uses the same engines and propellers to lift off vertically and to fly horizontally by reorienting its engines as shown.







Figure 17. The Boeing X-50A Dragonfly 21]

- Function Sharing Perform two or more discrete functions. A single part is designed so that it can perform different functions required by the different configurations of the device. The unmanned aerial vehicle X-50A Dragonfly (Fig. 17) being developed by Boeing offers the advantages of both helicopter and fixed wing aircraft. The wing of the X-50A consists of two segments that tilt independently and rotate about a fixed axis, acting like a helicopter. For fixed-wing flight the wings are held stationary and perpendicular to the fuselage like a plane.
- Furcate Change between two or more discrete stable states determined by boundary conditions. A product is designed with multiple stable states, and the transition between these states is defined by a set of boundary conditions imposed upon it. The common "slap bracelet" toy is an example of this facilitator; it is stable in its high-energy extended state until part of its cross-section is flattened, at which time it collapses to a lower-energy coiled state, shown sequentially in Figure 18.



Figure 18. Metal insert from a "slap bracelet" [22]

Storable Tubular Elastic Membranes (STEM) [9] have a similar bi-stable structural property that makes them capable of automatically varying their shape from a compact, packaged configuration to an expanded configuration. A STEM structure is shown in Figure 19.





Figure 19. STEM Structure [10]



Figure 20. Furcating Ball [23]



Figure 21. Venus Fly Trap [24]

Another interesting furcating product is a Phlat Ball, in Figure 20, which transforms into a disc and ball, both being stable states. It achieves this property by the use of its outer collapsible structure with flexible joints, a suction cup inside it to hold the two inner surfaces of the product in the disc configuration and a spring that helps the disc pop back to ball configuration when forces are applied on the sides of the disc.

The Venus Fly Trap can close its leaves fast enough to snare a fly, shown in Figure 21. It does this by releasing stored elastic energy in its leaves, reversing its curvature and allowing for a very quick reaction.

Table 1 is a list of all principles and facilitators that have been developed to date. While the applications of this theory can range across a wide range of products, this paper focuses on their application to a specific class of products: micro unmanned aerial vehicles.

III. **Research Objectives**

Micro aerial vehicles (MAVs) are used extensively in military operations to perform functions such as reconnaissance and surveillance. These situations in which MAVs are used often require rapid deployment of the device, frequently in imperfect conditions. Current MAVs typically are stored in either a disassembled state that requires a series of assembly operations or in an assembled state that requires a relatively large storage volume. A lightweight, quickly deployable, and easily operable alternative that can be stored in a smaller volume is desired. Improvements in these areas allow soldiers on the battlefield to function more efficiently while deploying the MAV and transporting it [25].

The wing usually requires the greatest storage volume per unit mass of the plane and therefore presents the greatest challenge when attempting to reduce its stored volume. The goal of this application is to use the transformation design methodology that is being developed to design a wing that can be used in such a capacity and is more easily stored than current designs.

Table 1. Transformation Principles and Facilitators [15]

Principles	Expand/Collapse	Change physical dimensions of object along and axis, in a plane, or in three dimensional space
	Expose/Cover	Expose/Cover a new surface to alter functionality
	Fuse/Divide	Make single functional device become two or more devices, at least one of which has its own distinct functionality defined by the state of the transformer or vice versa
	Common core structure	Compose devices with a core structure that remains the same, while the periphery reconfigures to alter the function of the device
	Composite	Form a single part from two or more parts with distinct functionality
	Conform with Structural Interfaces	Statically or dynamically constrain the motion of a component using structural interfaces
	Enclosure	Manipulate object in two or three dimensions in order to enclose a three dimensional space
	Flip	Perform separate functions based on orientation of the object
	Function Sharing	Perform two or more discrete functions
2	Furcation	Change between two or more discrete stable states determined by boundary conditions
Facilitators	Generic Connections	Employ internal or external connections (structural, power) that can be used by different modules to perform different functions or perform the same function in a different way
Щ. В	Interchangeable transmissions	Use multiple transmissions to produce different motions
	Material Flexibility	Change object dimensions with change in boundary conditions
	Modularity	Localize related functions utilizing common signal, material, and force flows into subsystems (modules) which are easily integrated into the device and may be interchangeable
	Nesting	Place an object inside another object wholly or partially wherein the internal geometry of the containing object is similar to the external geometry of the contained object
	Segmentation	Divide single contiguous part into two or more parts
	Shared Power Transmission	Transmit power from a common source to perform different functions in different configurations
	Shelling	Embed functional element in a device which performs a different function

A set of transformational design principles and facilitators have been developed that provide insights into the nature of transformers. The question that naturally arises is: how is this theory mapped to working concepts? The objectives of this research are to demonstrate: (1) the application of transformational design theory to a practical design problem, (2) the use of transformation theory in the context of existing design techniques, and (3) the application of metamodeling to optimize embodiment decisions that arise in transformational design. In order to accomplish these overarching goals, a design problem is proposed: the development and optimization of a transforming, easily storable wing with acceptable aerodynamic properties. The wing concept is designed using an experimental methodology which utilizes the transformational design principles and facilitators. The aims

of optimization are to maximize the lift to drag ratio of a small wing while maintaining a predefined performance envelope. Figure 22 shows the approach that will be taken to the problem.

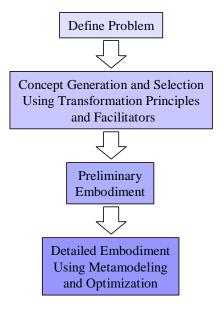


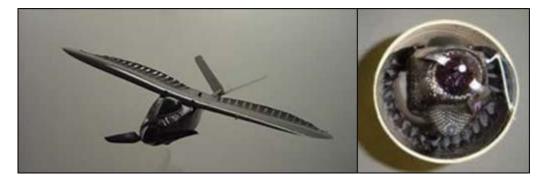
Figure 22. Problem Approach

After the development of several concepts, the premier idea is embodied and optimized. The design under consideration for optimization is a bi-stable wing design that is stable in its extended state and is also capable of maintaining a coiled configuration. This wing provides a suitable lifting surface when extended and can be rapidly returned to its coiled state for storage when not in use. Optimization is required to ensure that the wing will provide the greatest range possible using this design. To accomplish this, a metamodel is developed using data from a series of virtual experiments, because the analysis is otherwise very computationally intensive. Optimization is then performed to maximize the lift-to-drag ratio while maintaining other necessary flight characteristics. The final design is a quickly and easily deployable wing that greatly reduces storage volume with only slight degradation in flight performance.

A. Problem statement

The project to which our transformation methodology was applied is the development of an easily storable wing for Tactical Mini UAV (TACMAV) shown in Figure 23. Currently the TACMAV's wings curl underneath the fuselage for storage (Fig. 23(b)), but collapsibility is limited by the span on the wings since one wing impacts the underside of the other in this configuration. A separate tube is also required for storage in this configuration. A new

design is proposed that will improve this collapsing mechanism by reducing storage volume and eliminating the need for a separate storage container.



(a) (b) Figure 23. Tactical Mini UAV (TACMAV) (a) Deployed (b) Stored [26]

The completed design allows for the easy storage of the TACMAV without substantially degrading its performance with regard to lift and drag; these measures should not degrade by more than 20%. Additionally the wing is lightweight, quickly deployable, and easy to use. The design is otherwise open-ended.

B. Specifications of the TACMAV

This light battlefield surveillance MAV was developed by the Munitions Sector of the Air Force Research Laboratory (AFRL). It was designed for the United States Air Force (USAF) Battlefield Air Operations (BAO) unit. The TACMAV weighs less than 400 grams, and exemplifies the UAVs that are intended to replace current systems, which have similar capabilities but weigh two to three pounds. The TACMAV is powered by an electric motor and is hand-launched by the soldier. It uses a GPS/INS navigation system and is equipped with two TV cameras that provide real-time video transmitted to a PC-based ground station. The TACMAV is designed to complete hundreds of missions, but is inexpensive enough to be regarded as expendable. It is meant to provide the ability to be used by a covert land team. Some requirements of this MAV are to navigate, explore, and target, ultimately enhancing situational awareness, survivability, and mission success rates. The fact that this system will be deployed by foot soldiers in the field necessitates a lightweight, compact/collapsible structure that is easily deployed.

The TACMAV's airframe is constructed primarily of carbon fiber, resulting in a lightweight yet extremely robust structure. Carbon fiber also provides many characteristics that are beneficial to the production of MAV's. These include the ability to form complex shapes at relatively low cost, while retaining superior strength and stiffness [27]. Other specifications of the TACMAV are shown in Table 2.

Table 2: Existing TACMAV Specifications [28]

Length	61 cm (24 in)
Wingspan	51 cm (20 in)
Weight	0.38 kg (0.84 lb)
Speed (Range, Cruise)	10-50 mi/hr, 35 mi/hr
Ceiling	300 m (1000 ft)
Range	3 km (1.6 nm)
Endurance	18 min
Propulsion	Electric motor
Wing Area	93.5 in ²
Root Chord	6 in
Mean Aerodynamic Chord	4.2 in
Wing Thickness	0.025 in
Aspect Ratio	6.16

C. Concept generation and selection

The design theory that is being developed provides insights into the nature of transformation and a methodology that applies to design problems. Concept generation tools were adapted to help map the transformational principles and facilitators to working concepts. An extended mind mapping technique assists the designer in applying these ideas in practice (Fig. 24).

The traditional mind mapping approach is to write the problem to be solved in the center of a black sheet with a box around it. Ideas are generated to solve the central problem and are recorded in branches from the problem statement. As ideas are refined or spawn other ideas, these are connected to the parent idea on the map. The mind map helps organize the thoughts developed in a brainstorming session and helps the participants to create new ideas by branching off previous concepts [1].

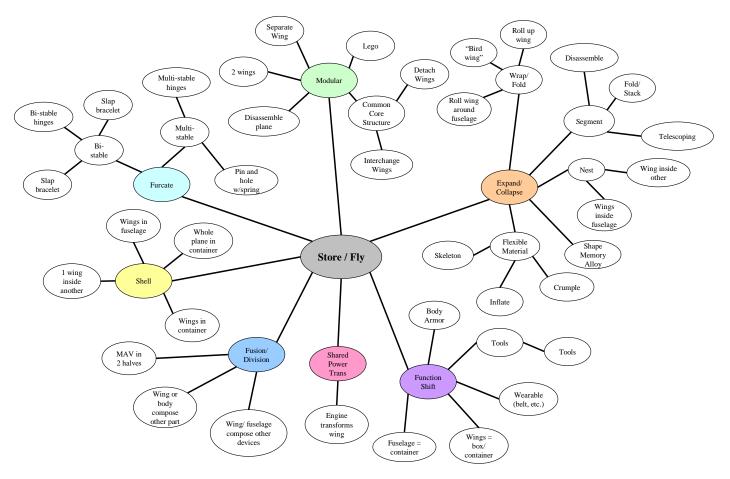


Figure 24. Transformational Mind Map

This technique was adapted to aid in the generation of transformers. The basic process is the same, with the transformational design problem in the center of the map. The problem is stated in the form of the two (or more) objectives of the transformer, in this case *Store / Fly*. The designer then chooses design principles and facilitators that may be of use in the development of a transition between these two and places these as branches around the problem statement. Ideas are then generated that are specific to each principle and placed around them as branches. As with a traditional mind map, each new idea can spawn new branches of its own. Special attention should be paid to interactions between the ideas attached to different principles since transformers frequently arise from a combination of different principles. A preliminary mind map for the storable MAV wing problem is shown in Figure 24. A collection of varying concepts were generated using this technique. Several of these are listed below:

- "Shape Memory Alloy" concept Uses shape memory alloy to expand the wing. For storage the wing can be folded to any convenient shape. It uses *Material Flexibility* to achieve *Expand/Collapse*.
- "Bird Wing" concept Is a spring-loaded wing that unfolds like the wing of a bird. This concept uses the
 principles Fuse/Divide and Expand/Collapse and the facilitators Segmentation and Conform with Structural
 Interface.
- "Inflatable Wing" concept Wing inflates for stiffness and can be deflated for storage. The impetus for this design was *Expand/Collapse* and it was facilitated by *Material Flexibility*.
- "Telescoping Wing" concept Telescopes from completely or partially inside the fuselage of the plane. The design requires the principles *Expand/Collapse* and *Fuse/Divide* and the facilitators *Segmentation* and *Nesting*.

The "Slap Bracelet" concept, shown in Figure 25, was chosen based on its ease of use, speed of deployment, weight, feasibility, novelty, and the manner in which it embodies the transformation principles. This concept combines ideas generated by the principle *expand/collapse* and the facilitator *furcate* to produce a collapsible, bistable wing similar to the common "slap bracelet" toy. This design has two stable configurations: (1) fully extended in the shape of a wing and (2) coiled alongside the fuselage. Each state is stable from an energy standpoint. The current stable configuration depends on the boundary conditions imposed on the wing; if the wing is straightened it will remain rigidly straight until part of its cross-section is flattened. This is accomplished by constructing the wing in such a way that it has a natural curvature in both the transverse and longitudinal directions. These curvatures oppose one another (one up and one down). Because the wing can only curve in one direction at a time, the wing is always at a high-energy state in one dimension. The wing, in this view, is always stressed in either the longitudinal or transverse direction. The transition between states occurs when the wing's cross section is flattened in one direction.

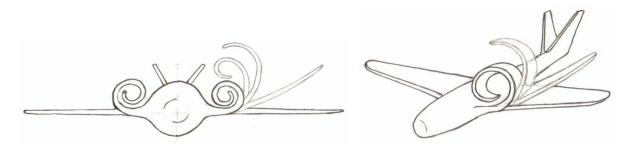


Figure 25. "Slap Bracelet" design concept

In the case of the wing concept, the transverse curvature is oriented downward at an angle to produce a thin airfoil shape, and the longitudinal curvature is oriented upward so that the wing rolls up next to the fuselage for storage.

D. Preliminary embodiment

The concept was further developed as a bi-stable collapsible wing made primarily of carbon fiber composite. Carbon fiber was chosen due to its high flexibility, high strength-to-weight ratio, and resistance to fatigue. Three variants of this concept were developed: (1) a wing made entirely of carbon fiber, (2) a wing made of carbon fiber with metal structural inlays, and (3) a wing made of carbon fiber with a bi-stable metal inlay.

The wing composed entirely of carbon fiber is to be constructed using two composite pieces with identical perimeter values. One of these components is curved upward longitudinally and the other is curved transversely in the other direction. The two components are laminated together to create a bi-stable structure which functions substantially like a slap bracelet [22].

Figure 26 shows the wing before assembly. The top wing will be formed with a longitudinal curvature and the bottom wing will be formed with a transverse curvature, as shown in the figure. The two parts of the wing will then be laminated to form the completed part, shown in Figure 27.

The wing composed of carbon fiber with metal structural inlays is formed using two composite pieces with identical perimeter values and one or more metal clock springs that span the length of the wing. The wing is laminated similarly to the all-composite design, with the clock springs contained between the two carbon fiber sheets. The carbon fiber sheets in this design both have transverse curvatures and the clock spring provides the force required to produce the longitudinal curvature and collapse the wing. Figure 28 shows the wing before assembly. The carbon fiber sheets are laminated together with the springs contained in between the two. The spring on the far side of the wing is shown in various stages of curling. Figure 29 shows the assembled wing.

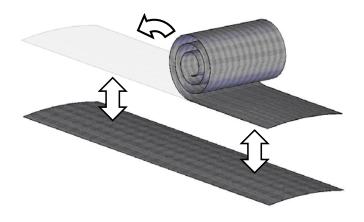


Figure 26. Composite Wing Concept (Pre-Assembly)

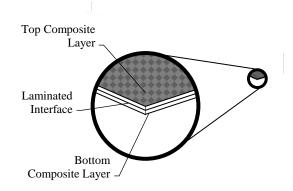


Figure 27. Carbon Fiber Wing (Assembled)

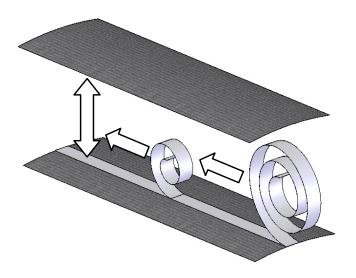


Figure 28. Clock Spring Concept (Pre-Assembly)

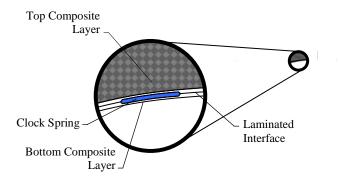


Figure 29. Clock Spring Concept (Assembled)

The third wing concept that was explored consists of a bi-stable metal element laminated between two carbon fiber sheets. This design differs from the previous design because the metal element in this wing is itself a bifurcating element. The carbon fiber wing surface therefore does not need to provide the transverse curvature necessary to maintain the wing in an extended position; the metal insert provides the required stiffness in both directions. Otherwise this design is substantially the same as the clock spring concept.

The three wings share the same finished geometry, as defined by the nature of the slap bracelet. The required form is a straight wing with a circle segment cross section. All three wings were modeled in SolidWorks in order to verify geometry and perform analysis.

E. Detailed Embodiment

A technique is required to make design decisions regarding the design parameters associated with the transforming wing concept. The development of analytical and experimental models is a necessary step in the development process. Due to the complex nature of the problem posed by the application of the design theory, a combination of analytically derived models and computational models and metamodels is proposed.

Because the design transforms, multiple states must be considered for model derivation. In the case of the collapsible wing, and for the purposes of demonstration, one state of the wing is analyzed: the in-flight configuration. Ultimately models developed for each state of the transformer will guide the designer in decisions

related to the interactions between states. They are necessary to provide an understanding of the whole system and will dictate the successful application of transformational design theory.

A computational fluid dynamics (CFD) model was used to simulate the performance of the wing in its extended state. The CFD model was computationally expensive to execute—a factor that made it unsuitable for analyzing iterative changes in the wing geometry. Instead, the CFD program was used to produce a metamodel of the system by application of a series of virtual experiments. Also, an approximate analytical model was derived to justify the selection of experimental points and to verify the results of the CFD analysis.

The goal of modeling the wing is to provide a mathematical equation(s) that can be used to optimize its flight parameters; the lift to drag ratio must be maximized with respect to a performance envelope defined by the original TACMAV performance. The variables of the design problem are summarized in Table 3. The CFD model and accompanying metamodel must accurately capture the relationships between these variables and the lift and drag of the system.

Table 3: Design Variables for Optimization

	Min	Max
Chord (in)	3	6
Camber (in)	0	0.45
Angle of Attack (deg)	0	14
Lift (lb)	1.83	N/A
Drag (lb)	N/A	0.188

1. Analytical Model

An analytical model is required in order to determine appropriate values for variables in our metamodel analysis, and to validate the CFD models. This model was developed using equations derived from Prandtl's lifting line theory and thin airfoil theory. While the results are not exact, they provide us with an estimation of lift and drag that can be compared with those obtained from our CFD models to ensure realistic results. Figure 30 shows the wing cross-section and details the system variables.

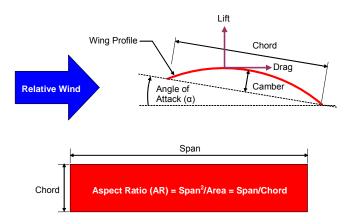


Figure 30. Thin Airfoil

To calculate the lift for a given wing, thin airfoil theory was used [28]. This theory assumes that the chord of an airfoil is much greater than its thickness, and restricts the design of an airfoil to low camber and angle of attack. These assumptions are particularly valid for our design as we are designing the wing as a thin sheet of carbon fiber. The theory further assumes that the velocity perpendicular to the surface of the wing is zero at all points. This allows for calculation of the local wind velocity at any point along the 2-dimensional wing profile. A further assumption is made that the wing is parabolic in shape rather than a circle segment. This simplifies the lift calculation and is acceptable because for wings with such low camber the difference in geometries is negligible. The lift for this wing design is calculated in Equation 1:

$$L = C_L \rho \frac{V^2}{2} A = \pi \left(\frac{2h}{c} + \alpha\right) \rho V^2 A \tag{1}$$

Drag was calculated using Prandtl's lifting line theory [29, 30]. This theory is based on the presence of trailing vortices created by the wing as it travels through the air. Equation 2 shows the analytical formula for drag. This equation calculates the total drag on the wing although effects in 3-dimensions, such as wing tip vortex drag and drag at the fuselage are estimated using efficiency factors for straight wing aircraft.

$$D = C_D \rho \frac{V^2}{2} A$$

$$= \left[C_{D,\text{min}} + \frac{AR \left(2\pi \left(\frac{2h}{c} + \alpha \right) \frac{1}{AR + 2} \right)^2}{\pi e} \right] \rho \frac{V^2}{2} A$$
(2)

Although these calculations provide us with a general guide that can be used to determine the rough accuracy of our CFD models, they are also very limited, necessitating a more accurate representation of our system. The most glaring limitation of these models is the unbounded nature of the formulas. In practice, wing performance degrades as parameters approach certain upper and lower bounds. Our model fails to capture this phenomenon. For example, in our model lift increases as angle of attack increases, regardless of the initial angle. In practice the angle of attack increases the lift up to the wing's stall angle, at which time the lift begins to decrease. If the angle continues to increase, the aircraft will lose lift very rapidly. Our model cannot account for these more complex interactions, and will increase unbounded in these situations. Similar incidents occur with the other parameters.

Another limitation of these models is in the assumptions made during calculation. Many simplifications were made to allow for calculation of our values in the absence of experimental data. Among these are a Fourier Series expansion used to estimate the value C_L , and the approximations for $C_{D,min}$ and e in the calculation of C_D [29, 30].

The model also makes assumptions about the type of flow and the shape of the wing. Due to the condition in thin airfoil theory that wind velocity perpendicular to the surface of the wing is zero, the model essentially assumes laminar flow. This does not occur in practice. The models also will only work for a straight-winged, thin airfoil design. Our wing design necessitates the same wing profile, but for other others such as swept wings or thicker airfoils, these estimates would be significantly less accurate. A CFD model was developed to more accurately represent the system.

2. Metamodel

Due to the complexity of the system, a basic analytical model could not be constructed with the desired accuracy; a more comprehensive CFD model was required to account for complexities in the physical system such as turbulence and flow separation. Therefore a series of analyses were performed in Cosmos FloWorks, and a metamodel of the system was created [31, 32]. The metamodel was then optimized for the maximum value of lift-to-

drag ratio. The constraints for modeling and optimization were determined based on the parameters of the original TACMAV, and system variables were determined from the original dimensions and varied within reasonable limits.

Design variables for this problem were the chord, camber, and angle of attack of the wing. These variables were chosen because together they define the lift and drag characteristics of the predefined wing profile. Since our basic wing shape was determined as a requirement of our design concept, these variables control the flight characteristics of our final optimized design. By varying these values, we can obtain the best performance available in our design space.

The original TACMAV has an average chord of 4.2 inches. During the CFD analysis, the chord was varied between 3 and 6 inches in step sizes of 1.5 inches. These values were chosen because we wish to maintain the current lift and drag values for the TACMAV, and we don't expect to drastically improve efficiency over the existing design. This is because the current TACMAV does not face the same profile constraints as our design and can therefore be constructed using more efficiently shaped airfoils. Since the length will not increase greatly in our design, we will likely require a similar chord to achieve equivalent values for lift and drag.

The range of 3 to 6 inches will allow us to explore the design space based on changes in the efficiency of the wing within reasonable limits. The upper bound was set in order to keep the rolled storage volume of the wing to a size compatible with the current aircraft fuselage, while the lower bound was chosen based on the developed equations because we are confident that adequate lift will not be produced below this level. The step size of 1.5 inches was selected based on computational time. It is anticipated that the angle of attack of the wing is the most important variable in this calculation, so some fidelity is sacrificed in the selection of values for chord and camber in order to increase the efficiency of the calculation.

Camber was varied between 0.00 and 0.45 inches in step sizes of 0.15 inch. These values were chosen based on the theoretical behavior seen in the analytical model of our system since higher cambers will likely produce drag well above the acceptable bound. As with chord, a coarse step size was chosen for camber because of the relative importance of angle of attack in the problem.

For a typical airfoil, lift increases linearly for small angles of attack, but lift decreases and drag increases very quickly as the wing approaches the stall angle. The angle of attack for our tests varied between 0° and 14° in step sizes of 2°. These values were chosen because the theoretical behavior of the system at higher angles of attack will

produce drag above the acceptable bound. Because the stall angle is an unknown quantity, the 2° step size is fine enough that we should capture the stall angle if it occurs below 14°.

The important constraints in our design optimization are wing length, velocity, minimum lift, and maximum drag. These values were determined based on the original design of the TACMAV [27], and were varied to allow for some performance degradation with our design. Generally speaking we wish to maintain or improve the performance of the plane.

The wing length was given an upper bound of 24 inches. The current wing length of the TACMAV is 20 inches, and since an increase in weight or size is undesirable, the length will not be allowed to increase more than 20% above the original value. Velocity was defined as 35 mph. This is the average cruising speed of the TACMAV under nominal conditions.

Minimum acceptable values for lift and drag were derived from the literature [27]. Experimental values for the lift and drag on the original TACMAV were provided and used to calculate the lower and upper bounds for lift and drag, respectively. The experimental lift value was decreased by 20% and the experimental drag was increased by 20% to allow for some minor degradation in performance for the new wing design. The final calculated values were $L_{min} = 1.83 \ lb$ and $D_{max} = 0.188 \ lb$. These values are still well within the performance requirements of the TACMAV and were used in our optimization algorithm.

Since the behavior of the system was unknown beforehand, we employed an exhaustive sampling approach to ensure that the metamodel captured output variations over small changes in the design space. We discretized each design variable, as shown in Table 4, and conducted a CFD analysis for each combination of design variable values. A total of 96 tests were conducted. This exhaustive experimental design is more computationally expensive than approaches such as fractional factorial designs [33], but yields better initial coverage of the design space. A comprehensive metamodel was constructed from this data from which optimal values could be obtained.

Table 4: CFD Input Variables

Chord (in)	Camber (in)	Angle of Attack (deg)
3.0	0	0
4.5	0.15	2
6.0	0.3	4
-	0.45	6

-	-	8
-	-	10
-	-	12
-	-	14

The model used to represent the physical system is Cosmos FloWorks running on the Solidworks platform. This program integrates directly with the CAD program to provide CFD analysis for 2 and 3-dimensional parts. 96 wing shapes were developed in Solidworks, each based on the original concept. 2-dimensional analysis was run to decrease computation time and expedite results; because our design is a straight wing with a constant cross-section, this is a valid approach. The wing analyzed was a unit span so the lift for the entire wing can be determined by multiplying by the total wingspan.

Using the collection of data from the complete set of tests, metamodels were constructed relating lift and drag to the input variables, chord, camber, and angle of attack. The models were fit using a second order regression. The derived formulas are shown in Equations 3 and 4, for drag, *D*, and lift, *L*, respectively.

$$D = 0.00837 - 0.00276 * c - 0.00150 * h$$

$$-0.00111 * \alpha + 0.00167 * c * h + 0.000284 * c * \alpha$$

$$-0.000545 * h * \alpha + 0.000219 * c^{2} + 0.0153 * h^{2}$$

$$+0.000101 * \alpha^{2}$$
(3)

$$L = 0.00489 + 0.00261 * c + 0.000245 * h$$

$$+ 0.00161 * \alpha + 0.0292 * c * h + 0.00101 * c * \alpha$$

$$+ 0.00492 * h * \alpha - 0.000209 * c^{2} - 0.185 * h^{2}$$

$$- 0.0000556 * \alpha^{2}$$
(4)

The model provided sufficiently accurate results based on the measured data to provide a mathematical model for the evaluation and optimization of the transforming wing. Table 5 shows the correlation coefficients for lift and drag, indicating close agreement of the model with the experimental CFD data. The points obtained are combinations of the variables shown in Table 4 which gives 96 points. The analytical model also verifies the metamodel, although much higher correlation was achieved at lower values of camber and angle of attack. This

result is expected since the effects that the model fails to capture (turbulence, flow separation, etc.) occur increasingly with higher cambers and steeper angles of attack. The analytical model does however show strong correspondence to the general trends exhibited in the metamodel, so it would appear to support the results of the CFD even in light of its limitations.

Table 5: Correlation Coefficients for Wing Models

	Lift	Drag
Correlation Coefficient	92.57%	93.53%

3. Optimization

Once a firm understanding of the system is gained from the model, optimization is required to produce the most efficiently performing design possible [34, 35]. This optimization guides the designer in making decisions that affect the different states of the wing. In this case the aerodynamic qualities of the wing are inspected and the efficiency of the wing is maximized with regard to the adjustable variables.

The goal of optimization was to maximize the lift to drag ratio while maintaining a minimum set of flight characteristics. The function chosen for optimization was *L/D* as defined by the metamodel. The optimization is formulated in Equations 5 and 6.

Maximize:

$$L/D = 0.584 - 0.945 * c - 0.164 * h$$

$$-1.45 * \alpha + 17.4 * c * h + 3.57 * c * \alpha$$

$$-9.04 * h * \alpha - 0.955 * c^{2} - 12.0 * h^{2}$$

$$-0.550 * \alpha^{2}$$
(5)

Subject to:

$$3" \le c \le 6"$$
 $0" \le h \le 0.45"$
 $0^{\circ} \le \alpha \le 14^{\circ}$
Lift $\ge 1.83 \text{ lb}$
Drag $\le 0.188 \text{ lb}$

The optimization problem was solved using the Generalized Reduced Gradient (GRG) solver in Excel [36] and the solutions were validated using a Sequential Quadratic Programming (SQP) algorithm in MATLAB [37]. Our

solution gives a lift to drag ratio of 13.4. The optimal values for chord, camber, and angle of attack are shown in Table 6.

Table 6: Optimized Wing Parameters

GRG Output				
Chord	Camber	AoA	Wingspan	L/D
6 in	0.247 in	3.59°	24 in	13.4

This result is similar to what is expected for such a wing profile and where the result is the combination of these values to maximize lift. These optimized design parameters can be used in the fabrication of a wing for an MAV such as the TACMAV where the wing can be further designed for collapsibility to store the MAV more efficiently.

F. Bistable wing fabrication

With the optimized design parameters and a method for fabricating this bistable wing, discussed in section 3.4, a 1:2 scaled down model of the wing is fabricated using carbon fiber. Figure 31(a) shows the collapsed wing required for storage and Figure 31(b) shows the in-flight configuration. This wing model is fabricated using glass reinforced unidirectional carbon fiber oriented 90° to the longitudinal direction of the bistable metallic inlay. The bistable metallic inlay is sandwiched between two 90° plies of carbon fiber. Use of carbon fiber creates additional planform area with great strength to weight ratio, required for good flight performance while not mitigating the bistable snap behavior of the metallic inlay.

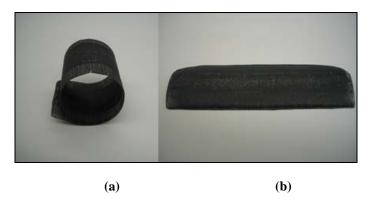


Figure 31. Bistable carbon fiber wing model (a) Collapsed; (b) In-flight extended

Behavior of bistable plates when embedded into systems such as wings is being studied, and prototypes are being fabricated [38]. However, the application of a bistable structure as a collapsible wing is a novel application presented in this paper. Figure 32 shows how 0° piles stacked with 90° piles of orthotropic materials can achieve a multi-stable laminated plate structure, with state change induced by thermal expansion/contraction. This specific laminated plate has two stable shapes shown in Figure 33(a) and (b).

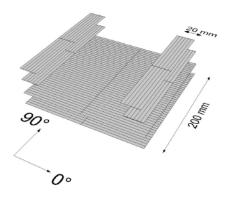


Figure 32. Stacking Sequence of a Transversely Reinforced Plate [38]

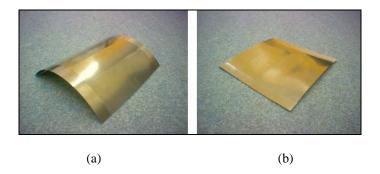


Figure 33. (a) Cylindrical Shape; (b) Flat Shape of the Transversely Reinforced Plate [38]

Other complex multi-stable structures are possible and can offer potentially more applications in aerospace. The benefit of the bistable or multi-stable ability is that an actuator is required only during the snap through process and no external energy is needed to hold the stable configurations. These designs enable a single structure to have two different geometric configurations.

IV. Conclusion

Transformation allows for the creation of devices that perform multiple functions where several single-state devices would otherwise be required. The design of these types of products provides numerous benefits, but requires

a systematic methodology for proper implementation. Currently, this methodology is being developed in the form of a set of principles and facilitators, and associated design tools for their application. While work in this area is ongoing, the design of a bifurcating collapsible wing, described in this paper, demonstrates preliminary results of a novel solution to a wing storage problem. A subset of principles and facilitators is presented and demonstrated using this practical example.

Given a general problem statement regarding the necessity for an easily storable wing, transformation theory and a modified mind-mapping technique is used to develop a wing concept that greatly reduces storage volume while maintaining performance. The new wing embodies the principles *collapse* and *furcate* that were developed as part of a transformational design methodology. In addition, it is easily and quickly deployable, and is lightweight enough to be manually transported. Using metamodeling and optimization techniques, the transforming wing concept is developed to perform to its maximum capabilities in the area of flight dynamics. An expansion of these models will allow further insights into the tradeoffs between performance levels in a single state as well as the tradeoffs between states that are inevitable in this process. A concurrent optimization process to make such determinations when comparing tasks performed in separate configurations will greatly benefit the completed design methodology.

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